The family of submanifolds of constant negative curvature in the Euclidean space included in orthogonal coordinate system

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Ealier J.D.Moore, K.Tenenblat, C.-L.Terng, F.Xavier and another authors considered isometric immersions of the domains of n-dimensional Lobachevsky space L^n into Euclidean space E^{2n-1}.

Now I want to consider the regular family of submanifolds F^n with constant negative curvature $K_0(F^n)$, which lies in some domain of E^{2n-1}. For example, we can suppose that this domain is a ball with the radius ρ. How large can be this ball, if $K_0 \leq -1$?

I have not answer on this question. But we take the additional condition. We suppose that this family can be included in the orthogonal coordinate system $u_1, ..., u_{2n-1}$ by such way, that the metric form of E^{2n-1} has the following representation

$$ds^2 = \sum_{i=1}^{n} H_i^2 du_i^2 + \sum_{\alpha=n+1}^{2n-1} H_\alpha^2 du_\alpha^2,$$

and coordinate submanifolds

$$u_{n+1} = const, ..., u_{2n-1} = const$$

are submanifolds F^n with constant negative curvature $K_0 \leq -1$. We remark that $K_0 = K_0(u_{n+1}, ..., u_{2n-1})$.

Bianchi considered similar system for n=2. He obtain the following expression for the metric of E^3

$$ds^2 = \cos^2\omega du_1^2 + \sin^2\omega du_2^2 + (\frac{1}{\sqrt{-K_0}}\frac{\partial\omega}{\partial u_3})^2 du_3^2.$$

Here we have the equation

$$\sum_{i=1}^{2} H_i^2 = 1.$$

Now we suppose that our coordinate system satisfies the condition

$$\sum_{i=1}^{n} H_i^2 = 1.$$

Such coordinate system we call Bianchi coordinate system.

Theorem 1. *Let there exist a regular Bianchi coordinate system with $K_0 \leq -1$ inside a ball D with radius ρ in the Euclidean space E^{2n-1}. Then*

$$\rho < \frac{\pi}{4}.$$

Theorem 2. *Bianchi coordinate systems locally exist in Euclidean spaces E^3 and E^5.*