Effect of point-contact transparency on the coherent mixing of Josephson and transport supercurrents

Gholamreza Rashedi[1] and Yuri A. Kolesnichenko[1,2]

[1]*Institute for Advanced Studies in Basic Sciences, 45195-159, Zanjan, Iran*

[2]*B.Verkin Institute for Low Temperature Physics Engineering of National Academy of Sciences of Ukraine, 47, Lenin avenue, 61103 Kharkov, Ukraine*

The influence of electron reflection on dc Josephson effect in a ballistic point contact with transport current in the banks is considered theoretically. The effect of finite transparency on the vortexlike currents near the contact and at the phase difference $\phi = \pi$, which has been predicted recently [Yu. A. Kolesnichenko, A. N. Omelyanchouk, and S. N. Shevchenko, Phys. Rev. B. **67** 172504 (2003)], is investigated. We show that at low temperatures even a small reflection on the contact destroys the mentioned vortexlike current states, which can be restored by increasing the temperature.

I. INTRODUCTION

The investigation of Josephson effect manifestations in different systems is continuing because of its importance for both basic science and industry. A point contact between two massive superconductors (S-c-S junction) is one of the possible Josephson weak links. A microscopic theory of the stationary Josephson effect in ballistic point contacts between conventional superconductors was developed in Ref. 1. Later, this theory was generalized for a pinhole model in ^{3}He,[2,3] for point contacts between "d-wave,"[4,5] and triplet superconductors.[6] The Josephson effect is the phase-sensitive instrument for the analysis of an order parameter in novel (unconventional) superconductors, where current-phase dependencies $I_J(\phi)$ may differ essentially from those in conventional superconductors.[4–6] In some cases the model with total transparency of the point contact does not quite adequately correspond to the experiment, and the electron reflection should be taken into account. The influence of electron reflection on the Josephson current in ballistic point contacts was first considered by Zaitsev.[7] He had shown that reflection from the contact not only changes the critical value of current, but also the current-phase dependence $I_J(\phi)$ $\sim \sin(\phi/2)$ at low temperature which has been predicted in Ref. 1. The current-phase dependence for small values of transparency, $D \ll 1$, is transformed to the $I_J(\phi) \sim \sin \phi$, similar to the planar tunnel junction. The effect of transparency for point contact between unconventional (d-wave) superconductors is studied in Refs. 8–10. The nonlocality of Josephson current in point contacts was investigated in Ref. 11. The authors of Ref. 11 concentrated on the influence of magnetic field on the zero voltage supercurrent through the junction. They found a periodic behavior in terms of magnetic flux and demonstrated that this anomalous behavior is a result of a nonlocality supercurrent in the junction. This observation was explained theoretically in Ref. 12. Recently an influence of transport supercurrent, which flows in the contacted banks and is parallel to the interface, to the Josephson effect in point contacts has been analyzed theoretically.[13] It was found that a nonlocal mixing of two superconducting

currents results in the appearance of two vortexlike current states in vicinity of the contact, when the external phase difference is $\phi \sim \pi$. The Josephson current through superconducting weak link is a result of quantum interference between order parameters with phase difference ϕ. Obviously, the finite reflection $R = 1 - D$ of electrons from the Josephson junction suppresses this interference and it must influence the vortexlike current states, which are predicted in Ref. 13. In this paper we study the effect of finite transparency on the current-phase dependence and distribution of the superconducting current near the ballistic point contact in the presence of homogeneous current states far from the contact. We show that at low temperatures ($T \to 0$) the electron reflection destroys the mentioned vortexlike current states even for a very small value of reflection coefficient $R \ll 1$. On the other hand we have found that as the temperature increases the vortices are restored and they exist for transparency as low as $D = \frac{1}{2}$ in the limit of $T \to T_c$. The organization of the rest of the paper is as follows. In Sec. II we describe the model of the point contact, quasiclassical equations for Green's functions, and boundary conditions. The analytical formulas for the Green functions are derived for a ballistic point contact with arbitrary transparency. In Sec. III we apply them to analyze a current state in the ballistic point contact. The influence of the transport current on the Josephson current and vice versa at the contact plane is considered. In Sec. IV we present the numerical results for the distribution of the current in the vicinity of the contact. We end in Sec. V with some conclusions.

II. FORMALISM AND BASIC EQUATIONS

We consider the Josephson weak link as a microbridge between thin superconducting films of thickness d. The length L and width $2a$ of the microbridge are assumed to be less than the coherence length ξ_0. On the other hand, we assume that L and $2a$ are much larger than the Fermi wavelength λ_F and use the quasiclassical approach. There is a potential barrier in the contact, resulting in a finite probability for the electron that is to be reflected back. In the banks of superconductors a homogeneous current with a supercon-
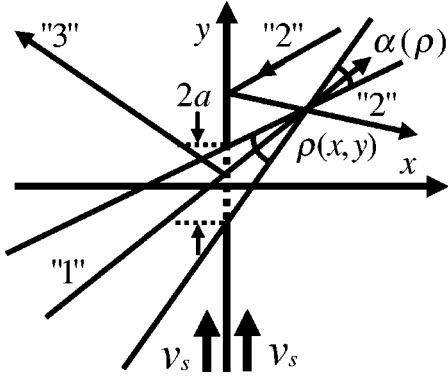
FIG. 1. Model of the contact as a slit in the thin insulating partition.

ducting velocity $\mathbf{v_s}$ flows parallel to the partition. We choose the y axis along $\mathbf{v_s}$ and the x axis perpendicular to the boundary; $x=0$ is the boundary plane (see Fig. 1). If the film thickness $d \ll \xi_0$ then in the main approximation in terms of the parameter d/ξ_0 the superconducting current depends on the coordinates in the plane of the film $\rho=(x,y)$ only. The superconducting current in the quasiclassical approximation,

$$\mathbf{j}(\rho,\mathbf{v_s})=-2\pi i e N(0)T\sum_{\omega_n}\langle\mathbf{v_F}g(\mathbf{v_F},\rho,\mathbf{v_s})\rangle_{\mathbf{v_F}}, \quad (1)$$

is defined by the energy-integrated Green's function

$$\hat{G}=\hat{G}(\omega_n,\mathbf{v_F},\rho,\mathbf{v_s})=\begin{pmatrix} g & f \\ f^+ & -g \end{pmatrix}, \quad (2)$$

which in the ballistic case satisfies the Eilenberger equation of the form[14,15]

$$\mathbf{v_F}\cdot\frac{\partial}{\partial\rho}\hat{G}+[\tilde{\omega}\hat{\tau}_3+\hat{\Delta},\hat{G}]=0, \quad (3)$$

with normalization condition, $g^2+ff^\dagger=1$. Here $N(0)$ is the density of states at the Fermi level, $\tilde{\omega}=\omega_n+i\mathbf{p}_F\cdot\mathbf{v_s}$, $\mathbf{v_F}$ and $\mathbf{p}_F$ are the electron velocity and momentum on the Fermi surface, $\omega_n=(2n+1)\pi T$ are the Matsubara frequencies, n is an integer number, $\mathbf{v_s}$ is the superfluid velocity and T is the temperature,

$$\hat{\Delta}=\begin{pmatrix} 0 & \Delta \\ \Delta^* & 0 \end{pmatrix} \quad$$

is the order parameter matrix, and $\hat{\tau}_3$ is the Pauli matrix. Equations (3) should be supplemented by the equation for the superconducting order parameter Δ,

$$\Delta(\rho,\mathbf{v_s},T)=2\pi\lambda T\sum_{\omega_n>0}\langle f(\mathbf{v_F},\rho,\mathbf{v_s})\rangle_{\mathbf{v_F}}, \quad (4)$$

where λ is the constant of pairing interaction and $\langle\cdots\rangle_{\mathbf{v_F}}$ is the averaging over directions of $\mathbf{v_F}$. As it was shown in Ref. 1 in the zero approximation in terms of the small parameter $a/\xi_0 \ll 1$ for a self-consistent solution of the problem it is not necessary to consider Eq. (4). The model in which the order

parameter is constant in the two half-spaces $\Delta(\rho,\mathbf{v_s},T)=\Delta(\mathbf{v_s},T)\exp[\mathrm{sgn}(x)(i\phi/2)]$ (ϕ is the phase difference between superconductors) can be used. In the same approximation the velocity $\mathbf{v_s}$ does not depend on the coordinates. Equation (4) enables us to calculate a spatial distribution of the order parameter $\Delta(\rho)$ in the next order approximation in terms of the parameter a/ξ_0. Solutions of Eqs. (3) should satisfy Zaitsev's boundary conditions (Ref. 7) across the contact $x=0,|y| \leq a$ and specular reflection condition for $x=0,|y| \geq a$. In addition, far from the contact, solutions should coincide with the bulk solutions. The Zaitsev boundary conditions at the contact can be written as[7–9,15]

$$\hat{d}^l=\hat{d}^r\equiv\hat{d}, \quad (5)$$

$$\frac{D}{2-D}\left[\left(1+\frac{\hat{d}}{2}\right)\hat{s}^r,\hat{s}^l\right]=\hat{d}\hat{s}^{l2}, \quad (6)$$

where

$$\hat{s}^r=\hat{G}_\omega^r(\mathbf{v_F},x=0)+\hat{G}_\omega^r(\mathbf{v_F}',x=0), \quad (7)$$

$$\hat{d}^r=\hat{G}_\omega^r(\mathbf{v_F},x=0)-\hat{G}_\omega^r(\mathbf{v_F}',x=0), \quad (8)$$

with $\mathbf{v_F}'$ being the reflection of $\mathbf{v_F}$ with respect to the boundary and D is the transparency coefficient of point contact. Indices l and r denote that the Green functions are taken at the left ($x=-0$) or right ($x=+0$) hand from the barrier. Similar relations also hold for $\hat{s}^l$ and $\hat{d}^l$. In general, D can be momentum dependent. For simplicity in our calculations we assumed that D is independent of the Fermi velocity direction.

III. CURRENT-PHASE DEPENDENCIES FOR JOSEPHSON AND TANGENTIAL CURRENTS

Making use of the solution of Eilenberger equations (3), we obtain the following expression for the current density (1) at the slit:

$$\mathbf{j}_{cont}=\mathbf{j}(x=0,|y|<a,\phi,\mathbf{v_s})=4\pi e N(0)v_F T\sum_{\omega>0}$$
$$\times\left\langle \hat{\mathbf{v}}\mathrm{Im}\left(\frac{\tilde{\omega}\Omega-i\eta D\Delta^2\sin\frac{\phi}{2}\cos\frac{\phi}{2}}{\Omega^2-\Delta^2 D\left(\sin\frac{\phi}{2}\right)^2}\right)\right\rangle_{\hat{\mathbf{v}}}, \quad (9)$$

where, $\Omega=\sqrt{\tilde{\omega}^2+\Delta^2}$, $\hat{\mathbf{v}}=\mathbf{v_F}/v_F$ is the unit vector, and $\eta=\mathrm{sgn}(v_x)$. We should require $\mathrm{Re}\,\Omega>0$, which fixes the sign of the square root to be $\mathrm{sgn}(\mathbf{p}_F\mathbf{v_s})$. In the case $\mathbf{v_s}\neq\mathbf{0}$ the current (9) has both $\mathbf{j}_J$ and $\mathbf{j}_y$ components. The tangential current $\mathbf{j}_y$ depends on the order parameter phase differences ϕ and is not equal to the transport current $\mathbf{j}_T$ on the banks. In other words the total current is not equal to the vector sum of Josephson and transport currents. For the case $\mathbf{v_s}=\mathbf{0}$, at the contact the tangential current is zero and the normal compo-
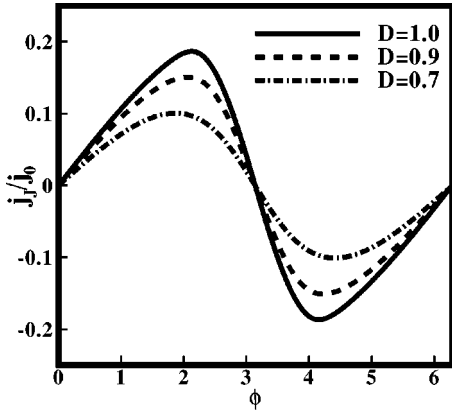
FIG. 2. Josephson current j_J vs phase ϕ for $T/T_c=0.1$, $q=0.5$, and $j_0=4\pi|e|N(0)v_FT_c$.

FIG. 3. Tangential current j_y vs phase ϕ for $T/T_c=0.1$ and $q=0.5$.

nent, i.e., the Josephson current, is as found for the finite transparent contact in Ref. 7. Detaching explicitly the Josephson current $\mathbf{j}_J$ and the spatially homogeneous (transport) current $\mathbf{j}_T$ that is produced by the superfluid velocity $\mathbf{v}_s$, we can write the current as the sum of three terms: $\mathbf{j}_J$, $\mathbf{j}_T$, and the "interference" current $\mathbf{j}_{int}$. Also we have

$$\mathbf{j}_{cont}=\mathbf{j}_J(\phi,D,\mathbf{v}_s)+\mathbf{j}_T(\mathbf{v}_s)+\mathbf{j}_{int}(\phi,D,\mathbf{v}_s). \qquad (10)$$

The interference current takes place in the vicinity of the contact, where both coherent currents $\mathbf{j}_J(\phi)$ and $\mathbf{j}_T(\mathbf{v}_s)$ exist (see also the following section). At first we consider the current density (9) for temperatures close to the critical temperature ($T_c-T\ll T_c$). From Eqs. (9) at the contact we obtain

$$\mathbf{j}_J(\phi,D,\mathbf{v}_s)=\tfrac{1}{2}AD\,\sin\phi\mathbf{e}_x, \qquad (11)$$

$$\mathbf{j}_T(\mathbf{v}_s)=-\tfrac{1}{3}Ak\mathbf{e}_y, \qquad (12)$$

$$\mathbf{j}_{int}(\phi,D,\mathbf{v}_s)=\tfrac{1}{3}AkD(1-\cos\phi)\mathbf{e}_y, \qquad (13)$$

where $A=\tfrac{1}{16}j_0(\Delta^2/T_c^2)$, $k=[14\mathsf{s}(3)/\pi^3](v_sp_F/T_c)$, and $\mathbf{e}_i$ is the unit vector in the i direction. This consideration shows how the current is affected by the interplay of Josephson and transport currents. At the contact the interference current $\mathbf{j}_{int}$ is antiparallel to $\mathbf{j}_T$ and if the phase difference $\phi=\pi$, $\mathbf{j}_{int}=-2D\mathbf{j}_T$. When there is no phase difference (at $\phi=0$), we obtain $j_{int}=0$. So at transparency values D up to $\tfrac{1}{2}$ the total tangential current at the contact flows in the opposite direction to the transport current. Thus, for such D in the vicinity of the contact, two vortices should exist. At arbitrary temperatures $T<T_c$ the current-phase relations can be analyzed numerically. In our calculations we define the parameter q in which $q=p_Fv_s/\Delta_0$ and $\Delta_0=\Delta(T=0,v_s=0)$. The value of q can be in the range $0<q<q_c$ and it's critical value q_c corresponds to the critical current in the homogeneous current state.[16] At $T=0$, $q_c=1$ and the gap Δ does not depend on q. In Figs. 2 and 3, we plot the Josephson and tangential currents at the contact as functions of ϕ at temperatures far from the critical (namely, $T=0.1T_c$) and for $q=0.5$ and different values of transparency D. Far from $\phi=\pi$, the tangential current is not disturbed by the contact, it tends to its value on the bank. The Josephson current-phase relation is the same as
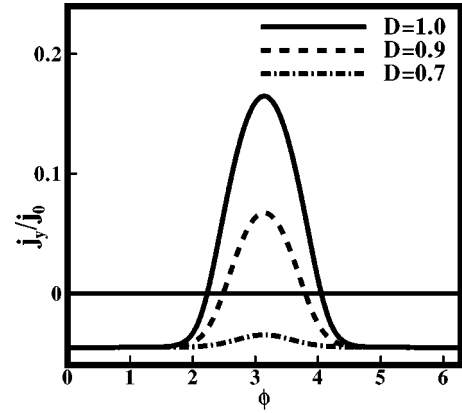
that when the transport current is absent. However, when ϕ tends to π, for the highly transparent contact ($D=1,0.9$) the tangential current becomes antiparallel to the bulk current. But for $D=0.7$ the interference current is strongly suppressed and the tangential current flows parallel to the bulk current. In Fig. 4, we plot $j_y(D)=j_T+j_{int}$ at $\phi=\pi$ for different temperatures. These plots show that by increasing the temperature a counterflow $j_y(D)<0$ exists in a wider interval of transparency $D_c(T)<D\leq1$ and $D_c(T\to T_c)\to\tfrac{1}{2}$. This numerical result coincides with analytical results (12) and (13).

IV. SPATIAL DISTRIBUTION OF THE CURRENT NEAR THE CONTACT

In this section we consider the spatial distribution of the current near the orifice. The superconducting current (1) can be written as

$$\mathbf{j}(\rho,\mathbf{v}_s)=-j_0\frac{T}{T_c}\sum_{\omega>0}\langle\hat{\mathbf{v}}\mathrm{Img}(\rho,\mathbf{v}_s)\rangle_{\hat{\mathbf{v}}_F}, \qquad (14)$$

where $j_0=4\pi|e|N(0)v_FT_c$. We should note that although the current (14) depends only on the coordinates in the film plane, the integration over velocity directions $\hat{\mathbf{v}}$ is carried out
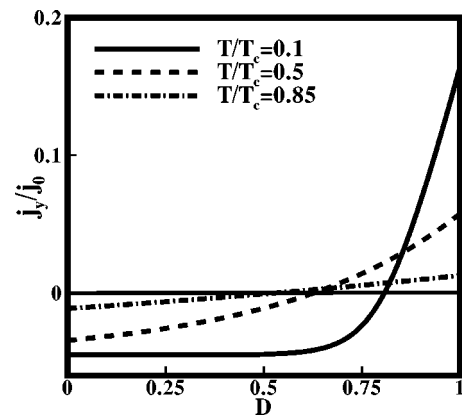

FIG. 4. Tangential current j_y vs the transparency D at $\phi=\pi$ and $q=0.5$.
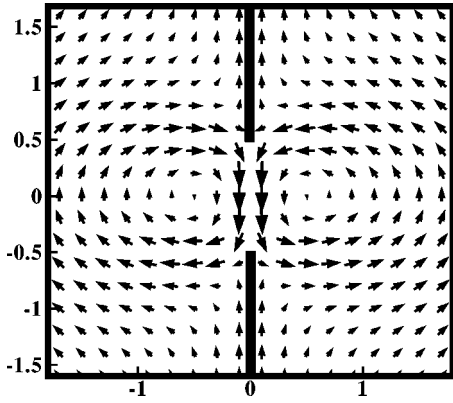
FIG. 5. Vector plot of the current for $\phi = \pi$, $q = 0.5$, $T/T_c = 0.1$, and $D = 0.95$. Axes are marked in units of the contact size a.

over all of the Fermi sphere as in a bulk sample. This method of calculation is correct only for specular reflection from the film surfaces when there is no backscattering after electron interaction with them. At a point $\rho = (x, y)$ all ballistic trajectories can be categorized as transit and nontransit trajectories (see, Fig. 1). For the transit trajectories "1" (their reflected counterparts marked by "3" in Fig. 1) a projection $\hat{\mathbf{v}}_\parallel$ of the vector $\hat{\mathbf{v}}$ to the film plane belongs to the angle at which the slit is seen from the point ρ, $\hat{\mathbf{v}}_\parallel \in \alpha(\boldsymbol{\rho})$, and for nontransit (marked by "2" in Fig. 1) $\hat{\mathbf{v}}_\parallel \notin \alpha(\boldsymbol{\rho})$. For transit trajectories Green's functions satisfy boundary conditions on both banks and at the contact. The nontransit trajectories should satisfy the specular reflection condition [or Zaitsev's boundary conditions (5) and (6) for $D = 0$ at $x = 0, |y| \geq a$]. Then for the current at $T_c - T \ll T_c$ we obtain an analytical formula

$$\mathbf{j}(\rho, \phi, D, \mathbf{v_s}) = j_c D \langle \sin \phi \hat{\mathbf{v}} \mathrm{sgn}(v_x) + k(1 - \cos \phi) \hat{\mathbf{v}} \hat{v}_y \rangle_{\hat{\mathbf{v}}_\parallel \in \alpha}$$

$$- j_c k \langle \hat{\mathbf{v}} \hat{v}_y \rangle_{\hat{\mathbf{v}}}, \qquad (15)$$

where, $j_c(T, \mathbf{v_s}) = [\pi |e| N(0) v_F / 8][\Delta^2(T, \mathbf{v}_s)/T_c]$. To illustrate how the current flows near the contact, we plot Figs. 5 and 6, for $\phi = \pi$ and temperatures much smaller than critical ($T/T_c = 0.1$), and for different values of transparency. At such value of the phase ϕ there is no Josephson current and
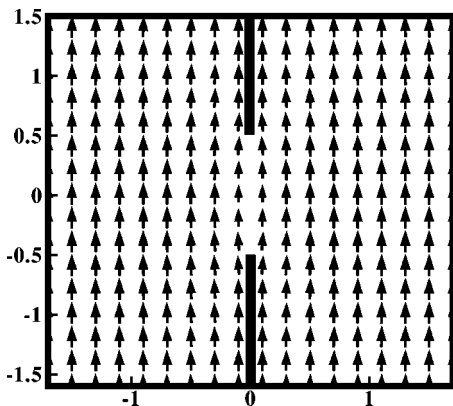

FIG. 6. Vector plot of the current for $\phi = \pi$, $q = 0.5$, $T/T_c = 0.1$, and $D = 0.7$.
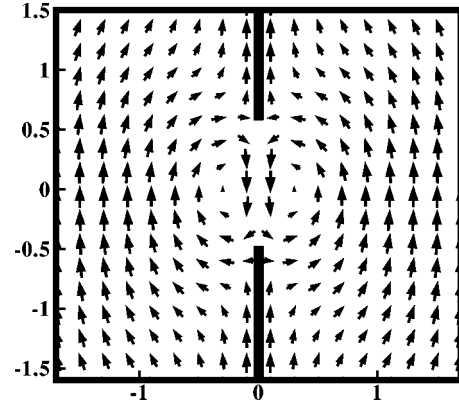

FIG. 7. Vector plot of the current for $\phi = \pi$, $q = 0.5$, $D = 0.7$, and $T/T_c = 0.85$.

at the large $D = 0.95$ the current is disturbed in such a way that there are two antisymmetric vortices close to the orifice (see Fig. 5). For such temperature at $D = 0.7$ the vortices are absent in Fig. 6. Near the critical temperature ($T/T_c = 0.85$) the vortexlike currents are restored for $D = 0.7$ (see Fig. 7). Far from the orifice (at the distances $l \sim \xi_0 \gg a$) the Josephson current is spread out and the current is equal to its value at infinity. Considering the current distributions and current-phase diagrams, we observed that (1) for fixed values of temperature and superfluid velocity, by decreasing the transparency the vortexlike current disappears at $D = D_c(T)$; $0.5 \leq D_c(T) < 1$ and (2) for intermediate values of transparency $D [D_c(T) < D < 1]$, by increasing the temperature the vortexlike currents, which were destroyed by the effect of electron reflection at the contact, may be restored. It is clear that both Josephson and interference currents are the result of the quantum interference between two coherent states. By decreasing the transparency the interference effect will be weaker and these two currents will decrease, while the transport current will remain constant. On the other hand, the presence of vortices depends on the result of competition between transport and interference current. Thus, by decreasing the transparency the tunneling and consequently the interference current will decrease and vortices may be destroyed [Eqs. (12) and (13)]. Similar to the case $D = 1$ in Ref. 13, at high values of transparency, the interference current can dominate the transport current and tangential current can be antiparallel to the transport current, thus the vortices appear. But for low transparency the tangential current will be parallel to the transport current and the vortices disappear. The second point is an anomalous temperature behavior of the effect. The vortices are the result of the coherent current mixing. One could expect that by increasing the temperature the vortices would disappear whereas, for intermediate values of transparency, by increasing the temperature the vortices will be restored. As considered in Figs. 6 and 7 for the transparency $D = 0.7$ the vortices at low temperature are absent but at high temperature they are present. In the plots for tangential current versus transparency, Fig. 4, we can observe this phenomenon (appearance of the counterflow near the contact at high temperatures). Usually superconducting currents are monotonic and descendant functions of temperature. Josephson and transport currents have this property, but about the tangential current j_y, the situation is totally differ-
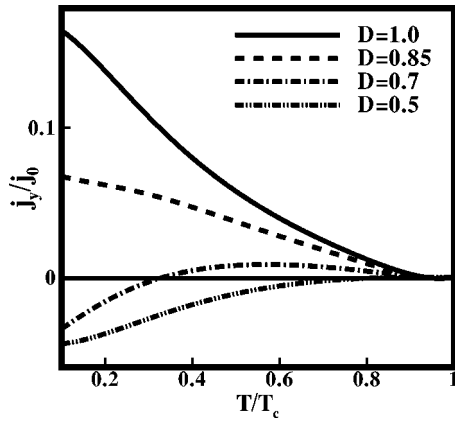
FIG. 8. Tangential current j_y vs the temperature T at $\phi = \pi$ and $q = 0.5$.

ent. At high values of transparency j_y has similar behavior to the two other currents, but at low and intermediate values of transparency at $\phi = \pi$ it has a nonmonotonic dependence on the temperature and this is the origin of the anomalous temperature behavior of vortices. As the temperature increases, the tangential current first increases and then decreases. In Fig. 8 we plotted the tangential current (interference + transport current) versus the temperature for different values of transparency. We observed that for intermediate values of transparency $0.5 < D < 1$, at low temperatures and $\phi = \pi$ the tangential current has anomalous dependence on the temperature. The reason for this dependence is that the interference current flows in the opposite direction to the transport current. This current is suppressed by the reflection, but with increasing temperature it decreases slower than the transport current. As a consequence of that, with increasing T the tangential current can change its sign and vortices can appear. We found that for low values of transparency, $0 < D < 0.5$, the interference current cannot dominate the transport current

and in addition the tangential current has the same direction as the transport current for any temperature $T < T_c$.

V. CONCLUSION

We have studied theoretically the stationary Josephson effect in the ballistic point contact with transport current on the banks in the model S-c-S taking into account the reflection of electrons from the contact. The contact is subject to two external factors: the phase difference ϕ and the transport current tangential to the boundary of the contact. As it was shown in Ref. 13, in the contact with direct conductivity at $\phi = \pi$ and near the orifice the tangential current flows in the opposite direction to the transport current, and there are two antisymmetric vortexlike structures. The transparency effect on the vortexlike currents has a central role in our paper. By decreasing the transparency, $D_c < D < 1$, the vortexlike current is destroyed. The critical value of $D = D_c(T)$ depends on the temperature T and $D_c(T \to 0) \to 1$, $D_c(T \to T_c) \to \frac{1}{2}$, so that we can never find a vortex for transparency values lower than $\frac{1}{2}$. This anomalous temperature behavior of the vortices is the result of nonmonotonic dependence of the interference current on the temperature. The principal possibility of the realization of the considered effect in an experiment was described in Ref. 13: A superconducting long thin-walled cylinder (with thickness of the wall d less than London penetration depth) with two cuts, such as the distance between them is smaller than coherence length ξ_0, is placed in magnetic field, which is parallel to the cylinder axis. A space between the cuts plays a role of the point contact. The phase difference ϕ is governed by the external magnetic flux. The transport current j_T flows through two large contacts at the ends of the cylinder.

ACKNOWLEDGMENT

[1] I.O. Kulik and A.N. Omelyanchouk, Fiz. Nizk. Temp. **4**, 296 (1978) [Sov. J. Low Temp. Phys. **4**, 142 (1978)].

[2] J. Kurkijärvi, Phys. Rev. B **38**, 11 184 (1988).

[3] S. Yip, Phys. Rev. Lett. **83**, 3864 (1999).

[4] M.H.S. Amin, A.N. Omelyanchouk, A.M. Zagoskin, Phys. Rev. B **62**, R6131 (2000).

[5] M. Fogelström, S. Yip, and J. Kurkijärvi, Physica C **294**, 289 (1998).

[6] R. Mahmoodi, S.N. Shevchenko, and Yu.A. Kolesnichenko, Fiz. Nizk. Temp. **28**, 262 (2002) [Sov. J. Low Temp. Phys. **28**, 184 (2002)].

[7] A.V. Zaitsev, Sov. Phys. JETP **59**, 1015 (1984).

[8] M.H.S. Amin, A.N. Omelyanchouk, and A.M. Zagoskin, Phys. Rev. B **63**, 212502 (2001); M.H.S. Amin, A.N. Omelyanchouk, S.N. Rashkeev, M. Coury, and A.M. Zagoskin, Physica B **318**, 162 (2002).

[9] Yu.S. Barash, A.V. Galaktionov, and A.D. Zaikin, Phys. Rev. B **52**, 665 (1995).

[10] T. Lofwander, V.S. Shumeiko, and G. Wendin, Supercond. Sci. Technol. **14**, R53 (2001).

[11] J.P. Heida, B.J. van Wees, T.M. Klapwijk, and G. Borghs, Phys. Rev. B **57**, R5618 (1998).

[12] V. Barzykin and A.M. Zagoskin, Superlattices Microstruct. **25**, 797 (1999); Urs. Lederman, Alban L. Fauchere, and Gianni Blatter, Phys. Rev. B **59**, R9027 (1999); M.H.S. Amin, A.N. Omelyanchouk, and A.M. Zagoskin, Low Temp. Phys. **27**, 616 (2001).

[13] Yu.A. Kolesnichenko, A.N. Omelyanchouk, and S.N. Shevchenko, Phys. Rev. B **67**, 172504 (2003).

[14] G. Eilenberger, Z. Phys. **214**, 195 (1968).

[15] W. Belzig, F.K. Wilhelm, C. Bruder, G. Schon, and A. Zaikin, Superlattices Microstruct. **25**, 1251 (1999).

[16] J. Bardeen, Rev. Mod. Phys. **34**, 667 (1962).